1. A woodwork teacher measures the width, w mm, of a board. The measured width, x mm, is normally distributed with mean w mm and standard deviation 0.5 mm.

(a) Find the probability that x is within 0.6 mm of w.

(2)

The same board is measured 16 times and the results are recorded.

(b) Find the probability that the mean of these results is within 0.3 mm of w.

(4)

Given that the mean of these 16 measurements is 35.6 mm,

(c) find a 98% confidence interval for w.

(4)
(Total 10 marks)

2. The heights of a random sample of 10 imported orchids are measured. The mean height of the sample is found to be 20.1 cm. The heights of the orchids are normally distributed.

Given that the population standard deviation is 0.5 cm,

(a) estimate limits between which 95% of the heights of the orchids lie,

(3)

(b) find a 98% confidence interval for the mean height of the orchids.

(4)

A grower claims that the mean height of this type of orchid is 19.5 cm.

(c) Comment on the grower's claim. Give a reason for your answer.

(2)
(Total 9 marks)

3. A random sample of the daily sales (in £s) of a small company is taken and, using tables of the normal distribution, a 99% confidence interval for the mean daily sales is found to be

(123.5, 154.7)

Find a 95% confidence interval for the mean daily sales of the company.

(Total 6 marks)

4. A machine produces metal containers. The weights of the containers are normally distributed. A random sample of 10 containers from the production line was weighed, to the nearest 0.1 kg, and gave the following results

49.7, 50.3, 51.0, 49.5, 49.9

50.1, 50.2, 50.0, 49.6, 49.7.

(a) Find unbiased estimates of the mean and variance of the weights of the population of metal containers.

(5)

The machine is set to produce metal containers whose weights have a population standard deviation of 0.5 kg.

(b) Estimate the limits between which 95% of the weights of metal containers lie.

(4)

(c) Determine the 99% confidence interval for the mean weight of metal containers.

(5)

(Total 14 marks)

5. The drying times of paint can be assumed to be normally distributed. A paint manufacturer paints 10 test areas with a new paint. The following drying times, to the nearest minute, were recorded.

82, 98, 140, 110, 90, 125, 150, 130, 70, 110.

(a) Calculate unbiased estimates for the mean and the variance of the population of drying times of this paint.

(5)

Given that the population standard deviation is 25,

(b) find a 95% confidence interval for the mean drying time of this paint.

(5)

Fifteen similar sets of tests are done and the 95% confidence interval is determined for each set.

(c) Estimate the expected number of these 15 intervals that will enclose the true value of the population mean μ.

(2)
(Total 12 marks)

6. A computer company repairs large numbers of PCs and wants to estimate the mean time to repair a particular fault. Five repairs are chosen at random from the company's records and the times taken, in seconds, are

205 310 405 195 320.

(a) Calculate unbiased estimates of the mean and the variance of the population of repair times from which this sample has been taken.

(4)

It is known from previous results that the standard deviation of the repair time for this fault is 100 seconds. The company manager wants to ensure that there is a probability of at least 0.95 that the estimate of the population mean lies within 20 seconds of its true value.

(b) Find the minimum sample size required.

(6)
(Total 10 marks)

7. Kylie regularly travels from home to visit a friend. On 10 randomly selected occasions the journey time x minutes was recorded. The results are summarised as follows.

$\sum x = 753, \quad \sum x^2 = 57\,455.$

(a) Calculate unbiased estimates of the mean and the variance of the population of journey times.

(3)

After many journeys, a random sample of 100 journeys gave a mean of 74.8 minutes and a variance of 84.6 minutes^2.

(b) Calculate a 95% confidence interval for the mean of the population of journey times.

(5)

(c) Write down two assumptions you made in part (b).

(2)
(Total 10 marks)

8. A random sample of 30 apples was taken from a batch. The mean weight of the sample was 124 g with standard deviation 20 g.

(a) Find a 99% confidence interval for the mean weight μ grams of the population of apples. Write down any assumptions you made in your calculations.

(6)

Given that the actual value of μ is 140,

(b) state, with a reason, what you can conclude about the sample of 30 apples.

(2)
(Total 8 marks)

1. (a) $E \sim \mathrm{N}(0, 0.5^2)$

or

$X \sim \mathrm{N}\left(w, 0.5^2\right)$

$\mathrm{P}(|E| < 0.6) \quad = \mathrm{P}\left(|Z| < \frac{0.6}{0.5}\right)$

or

$\mathrm{P}\left(|X - w| < 0.6\right) = \mathrm{P}\left(|Z| < \frac{0.6}{0.5}\right)$ M1

$= \mathrm{P}(|Z| < 1.2)$

$= 2 \times 0.8849 - 1 = 0.7698$ awrt **0.770** A1 2

<u>Note</u>

1st M1 for identifying a correct probability (they must have the 0.6) and attempting to standardise. Need | |. This mark can given for 0.8849 – 0.1151 seen as final answer.

1st A1 for awrt 0.770. NB an answer of 0.3849 or 0.8849 scores M0A0 (since it implies no | |)

M1 may be implied by a correct answer

(b) $\bar{E} \sim \mathrm{N}\left(0, \frac{1}{64}\right)$

or

$\bar{X} \sim \mathrm{N}\left(w, \frac{0.5^2}{16}\right)$ M1

$\mathrm{P}\left(\left|\bar{E}\right| < 0.3\right) \quad = \mathrm{P}\left(\left|Z\right| < \frac{0.3}{\frac{1}{8}}\right)$

or

$\mathrm{P}\left(\left|\bar{X} - w\right| < 0.3\right) = \mathrm{P}\left(\left|Z\right| < \frac{0.3}{\frac{1}{8}}\right)$ M1, A1

$= \mathrm{P}(\left|Z\right| < 2.4)$

$= 2 \times 0.9918 - 1 = 0.9836$ awrt **0.984** A1 4

Note

1st M1	for a correct attempt to define $\bar{E}$ or $\bar{X}$ but must attempt $\frac{\sigma^2}{n}$. Condone labelling as *E* or *X* This mark may be implied by standardisation in the next line.
2nd M1	for identifying a correct probability statement using $\bar{E}$ or $\bar{X}$. Must have 0.3 and | |
1st A1	for correct standardisation as printed or better
2nd A1	for awrt 0.984

The M marks may be implied by a correct answer.

Sum of 16, not means

1st M1	for correct attempt at suitable sum distribution with correct variance (= 16 × $\frac{1}{4}$)
2nd M1	for identifying a correct probability. Must have 4.8 and | |
1st A1	for correct standardisation i.e. need to see $\frac{4.8}{\sqrt{4}}$ or better

	(c)	$35.6 \pm 2.3263 \times \frac{1}{8}$	M1 B1	
		(**35.3, 35.9**)	A1, A1	4

Note

M1 for $35.6 \pm z \times \frac{0.5}{\sqrt{16}}$

B1 for 2.3263 or better. Use of 2.33 will lose this mark but can still score ¾

1st A1 for awrt 35.3

2nd A1 for awrt 35.9

[10]

2.	(a)	Limits are 20.1 ± 1.96 × 0.5	M1 B1	
		<u>**(19.1, 21.1)**</u>	A1cso	3

Note

M1 for $20.1 \pm z \times 0.5$. Need 20.1 and 0.5 in correct places with no $\sqrt{10}$

B1 for $z = 1.96$ (or better)

A1 for awrt 19.1 <u>and</u> awrt 21.1
but must have scored both M1 and B1

[Correct answer only scores 3/3]

	(b)	98% confidence limits are	M1	
		$20.1 \pm 2.3263 \times \frac{0.5}{\sqrt{10}}$	B1	
		<u>**(19.7, 20.5)**</u>	A1A1	4

Note

M1 for $20.1 \pm z \times \frac{0.5}{\sqrt{10}}$ need to see 20.1, 0.5 and $\sqrt{10}$ in correct places

B1 for $z = 2.3263$ (or better)

1st A1 for awrt 19.7

2nd A1 for awrt 20.5

[Correct answer only scores M1B0A1A1]

(c) The growers claim is not correct — B1

Since 19.5 does not lie in the interval (19.7, 20.5) — dB1 — 2

<u>Note</u>

1st B1 for rejection of the claim. Accept "unlikely" or "not correct"

2nd dB1 Dependent on scoring 1st B1 in this part for rejecting grower's claim for an argument that supports this. Allow comment on <u>their</u> 98% CI from (b)

[9]

3. $\bar{x} - \frac{1}{2}(123.5 + 154.7) - \underline{139.1}$ — B1

2.5758 — B1

"their 2.5758" $\frac{\sigma}{\sqrt{n}}$ = 154.7 – 139.1 = 15.6 — M1

AWRT 1.96 — B1

"their 1.96" $\frac{\sigma}{\sqrt{n}} = \frac{15.6 \times 1.96}{2.5758}$ = (11.87...) — M1

So 95% C.I. = 139.1 ± 11.87... = (127.22..., 150.97...) — AWRT <u>(127, 151)</u> — A1 — 6

1st B1 for mean = 139.1 only

1st M1 for UL – mean or mean – LL set equal to z value times standard error or some equivalent expression for standard error. Follow through their 2.5758 provided a z value.

May be implied by $\frac{\sigma}{\sqrt{n}}$ = 6.056... (N.B. $\frac{15.6}{2.3263}$ = 6.705...]

Condone poor notation for standard error if it is being used correctly to find CI.

2nd M1 for full method for semi-width (or width) of 95% interval Follow through their z values for both M marks

N.B. Use of 2.60 instead of 2.5758 should just lose 2nd B1 since it leads to AWRT (127, 151)

[6]

4. (a) $\overline{X} = \frac{500}{10} = 50$ M1 A1

$S^2 = \frac{1}{9}(25001.74 - \frac{500^2}{10}) = 0.193$ awrt 0.193 M1 A1 A1 5

(b) Limits are 50 ± 1.966 B1 1.96 M1 B1

$= (49.02, 50.98)$ awrt 49(0), 51(0) A1 A1 4

(c) Confidence interval is

$(50 - 2.5758 \times \frac{0.5}{\sqrt{10}}, 50 + 2.5758 \times \frac{0.5}{\sqrt{10}})$ B1 2.5758 M1 B1 A1ft

$= (49.59273, 50.40727\ldots)$ awrt 49.6, awrt 50.4 A1 A1 5

Use of estimate in (a) in (b) AND (c) assume MISREAD

[14]

5. (a) $\hat{\mu} = \frac{82 + 98 + 140 + 110 + 90 + 125 + 150 + 130 + 70 + 110}{10}$ M1

$= 110.5$ A1

$\hat{\sigma} = \frac{1}{9}(128153 - 10 \times 110.5^2)$ B1

128153 M1

$= 672.28$ (AWRT 672) A1 5

(b) 95% confidence limits are (condone use of 5 instead of 25) M1

(for 1.96) B1

A1ft

$110.5 \quad \pm 1.96 \times \frac{25}{\sqrt{10}}$ A1 A1 5

95% conf. lim. = AWRT(95, 126)

(c) Number of intervals = $\frac{95}{100} \times 15$ M1

$= 14.25$ (allow 14 or 14.3 if method is clear) A1 2

[12]

6.	(a)	Let X represent repair true		
		$\therefore \sum x = 1435 \therefore \bar{x} = \frac{1435}{5} = \underline{287}$	B1	
		$\sum x^2 + = 442575 \;\therefore s^2 = \frac{1}{4}\left\{442575 - \frac{1435^2}{5}\right\}$	M1 A1	
		$= \underline{7682.5}$	A1	4
	(b)	$\mathrm{P}(\lvert \mu - \hat{\mu} \rvert) < 20 = 0.95$	M1	
		Use of 10,20 as 40 with their σ & $\sqrt{n}$		
		$\therefore \frac{20}{\sigma/\sqrt{n}} = 1.96$	B1	
		$\therefore n = \frac{1.96^2 \sigma^2}{20^2} = \frac{1.96^2 \sigma^2}{400} = \underline{96.04}$	M1 A1	
		Solving for n		
		$\therefore$ Sample size ($\geq$) 97 required	A1	6

[10]

7.	(a)	$\bar{x} = 75.3$	B1	
		$s^2 = \frac{1}{9}\left\{57455 - \frac{753^2}{10}\right\}$	M1	
		$= 83.7\dot{8},\; 83\frac{71}{90},\; 83.8$	A1	3
		awrt 83.8		
	(b)	$74.8 \pm 1.96\sqrt{\frac{84.6}{100}}$		
		1.96	B1	
		any z value, may use 75.3, 83.8 for M	M1	
			A1ft on z only	
		(73.0, 76.6)	A1, A1	5
		awrt 73.0,76.6		

	(c)	Journey times independent Sample large enough to use central limit theorem Same distribution / population	any 2 B1, B1	2
				[10]
8.	(a)	X ~ N (124, 20^2) or $\overline{X}$ ~ (124, $\frac{20^2}{30}$) or assume σ^2 estimated by s^2 or CLT, vals.	B1, B1	
		$\bar{x} \pm 2.5758 \frac{\sigma}{\sqrt{n}} = 124 \pm 2.5758 \frac{20}{\sqrt{30}}$ 2.5758, formula + attempt, all correct & 2.58,2.576	B1 M1 A1	
		= 124 ± 9.405 = (115.133)	3 sf A1	6
	(b)	140 is not in confidence interval	M1	
		Underweight apples chosen or Sample **may** not be representative/ **may** be biased	Any one A1∫	2
				[8]

1. The first two parts of this question proved quite challenging for many candidates who were unable to handle the modulus. Many simply missed the significance of the phrase "…is within…" and others misinterpreted the ranges dividing by 2. Those who did interpret the question correctly usually had few problems in calculating the required probabilities although there were some errors with the variance in part (b).

The confidence interval in part (c) was answered very well by most candidates and many correct solutions were seen. Few failed to use $z = 2.3263$ and only a small minority used an incorrect standard error.

2. It is often the case when we set questions like part (a) that many candidates simply calculate a 95% confidence interval for the mean and the same happened here. They then repeated the technique in part (b), usually with the correct z value, and there were many fully correct confidence intervals found here. Some candidates were not prepared to make a decision in part (c) and hedged their comments, possibly because 19.5 was "in" the answer to part (a) but not in the confidence interval in (b). The examiners required a clear rejection of the grower's claim, based on the fact that 19.5 was outside the 98% confidence interval and a good number of candidates gained both marks for these two simple statements.

3. Whilst many candidates scored well on this question, others barely knew how to start and would often score no more than a couple of marks for looking up relevant values in the tables. Despite this, few answered the question well with poor notation ($\frac{\sigma^2}{n}$, σ or worse were often seen used for standard error) and an absence of words to explain what they were doing meaning that many solutions were a jumble of figures and symbols from which the examiners could sometimes see the correct answer emerging.

4. Part (a) was almost always completely correct, but parts (b) and (c) elicited the usual mistakes involving incorrect variances. In part (b) it was not unusual to see $50 \pm 1.96 \frac{0.5}{\sqrt{10}}$

5. The mean was almost always correct and the majority of the candidates knew how to find an unbiased estimate of the variance. Most knew how to find 95% confidence limits although a few used 1.6449 and some thought the formula was $1.96 \pm \frac{\sigma}{\sqrt{n}}$. Occasionally the standard deviation of 25 was misread as a variance. The final part caused some confusion. Many students carried out the required calculation but some started to find the width of the interval and others did not attempt this part.

6. Part (a) was usually well answered by most of the candidates. Many candidates did not really understand how to tackle part (b) and of those that did many did not interpret the phrase ‘lies within’ correctly.

7. Candidates generally did either very well with this question or very badly. Many managed to find the mean but failed to find an unbiased estimate of the variance. Many different and some times very odd formulae were used but $\frac{1}{10}(57455 - \frac{753^2}{10})$ was fairly common. In part (b) the z value was often correct but candidates then went on to use $\frac{84.6}{10}$ rather than $\sqrt{\frac{84.6}{100}}$. In part (c) it was usually recognised that the central limit theorem could be invoked but few other correct assumptions were given.

8. Assumptions were often left out or wrong. A number of candidates mentioned the Central Limit Theorem and nothing more. The formula was applied well, although the majority left their answers to more than 3 significant figures. In part (b) many scored only one mark as they concluded with too much certainty.